II PU MATHEMATICS - 2024

35 (NS)

(English Version)

Instructions :

1. *The question paper has five Parts namely A, B, C, D and E. Answer **all** the parts.*
2. *Part-A has **15** multiple choice questions and **5** fill in the blank questions.*
3. *For Part-A questions, only the first written answers will be considered for evaluation.*
4. *Use the graph sheet for question on Linear Programming Problem in Part-E.*

PART – A

I. Answer **all** the multiple choice questions : **($15 \times 1 = 15$)**

1) The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (1, 2), (2,3), (3,3)\}$ is

   a) Reflexive
   b) Reflexive and Symmetric
   c) Reflexive and Transitive
   d) Symmetric and Transitive

2) If $f: Z \rightarrow Z$, where Z is the set of integers is defined as $f(x) = 3x$ then

   a) f is both one-one and onto
   b) f is many one and onto
   c) f is one-one but not onto
   d) f is neither one-one nor onto

3) The principal value branch of $\sin^{-1} x$ is

   a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
   b) $(0, \pi)$
   c) $[0, \pi]$
   d) $[0, 2\pi]$

4) If $A=[a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij}=\frac{i}{j}$ then A is

a) $\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & \frac{1}{2}\\ 2 & 1\end{bmatrix}$

c) $\begin{bmatrix}0 & 2\\ \frac{1}{2} & 0\end{bmatrix}$ d) $\begin{bmatrix}1 & 2\\ \frac{1}{2} & 1\end{bmatrix}$

5) If A is an invertible matrix of order 2×2 $\det(A)=5$ then $\det(A^{-1})$ is equal to

a) 5 b) $\frac{1}{25}$

c) $\frac{1}{5}$ d) 25

6) The function $f: R \rightarrow R$ defined as $f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x. For what values of x in the interval $2<x<5$ given below $f(x)$ is not differentiable?

a) 2 and 5 b) 3 and 5

c) 4 and 5 d) 3 and 4

7) If $y=\sin(x^2+5)$ then $\frac{dy}{dx}$ is

a) $\cos(x^2+5)$ b) $-2x\cos(x^2+5)$

c) $\cos(x^2+5)(2x+5)$ d) $2x\cos(x^2+5)$

8) The maximum value of the function $f(x)=x$, $x \in (1, 2)$ is

a) 1 b) do not have maximum value

c) 3 d) 2

9) $\int \sec x(\sec x+\tan x)dx$ is

a) $\sec^2 x+\tan x+c$ b) $\sec x+\tan x+c$

c) $\sec x-\tan x+c$ d) $-\tan x-\sec x+c$

10) $\int e^x (\sin x + \cos x) dx$ is equal to

a) $e^x \cos x + c$
b) $e^x \tan x + c$
c) $e^x \sin x + c$
d) $-e^x \cos x + c$

11) The projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ along x-axis is

a) 1
b) 3
c) 7
d) 0

12) The unit vector in the direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is

a) $\dfrac{\hat{i} - \hat{j} - 2\hat{k}}{6}$
b) $\dfrac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$
c) $\dfrac{\hat{i} - \hat{j} + 2\hat{k}}{6}$
d) $\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

13) If a line makes 90°, 135°, 45° with the x, y and z axes respectively then direction cosines are

a) $0, \dfrac{-1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$
b) $0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$
c) $1, \dfrac{-1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$
d) $1, \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

14) Which of the following is a non negative constraints in a Linear Programming Problem?

a) $x \geq 0,\ y \leq 0$
b) $x \leq 0,\ y \leq 0$
c) $x \geq 0,\ y \geq 0$
d) $x \leq 0,\ y \geq 0$

15) If two cards are drawn without replacement from a pack of 52 playing cards then the probability that both the cards are black is

a) $\dfrac{1}{26}$
b) $\dfrac{1}{4}$
c) $\dfrac{25}{104}$
d) $\dfrac{25}{102}$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket : **(5 × 1 = 5)**

($0, \; 1, \; 2, \; 3, \; \frac{1}{2}, \; 6$)

16) The value of $\sin(\text{cosec}^{-1}2)$ is ________.

17) If A is a square matrix of order 2×2 and $|A| = 8$ then $\left|\frac{1}{2}A\right|$ is ________.

18) The order of the differential equation $\frac{d^3y}{dx^3} + y^2 + e^{\frac{dy}{dx}} = 0$ is ________.

19) Two lines with direction ratios 1, 3, 5 and 2, K, 10 are parallel then the value of K is ________.

20) If F is an event of the sample space S and $P(F) \neq 0$ then $P(S/F)$ is ________.

PART – B

Answer **any six** questions : **(6 × 2 = 12)**

21) Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x, \; -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$.

22) Find the equation of the line joining the points (3, 1) and (9, 3) using determinants.

23) If $2x + 3y = \sin y$ then find $\frac{dy}{dx}$.

24) The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

25) Find the interval in which the function f given by $f(x)=2x^2-3x$ is decreasing.

26) Find $\int \frac{x}{(x+1)(x+2)}dx$.

27) Evaluate $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$.

28) Consider two points P and Q with position vectors $\vec{OP} = 3\vec{a} - 2\vec{b}$ and $\vec{OQ} = \vec{a} + \vec{b}$. Find the position vector of a point R which divides the line joining P and Q internally in the ratio 2 : 1.

29) Find the angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$.

30) Two coins are tossed once, where the events E and F are defined as

E : Tail appears on one coin

F : One coin shows Head

Find $P(E/F)$.

31) Let A and B be 2 independent events such that $P(A)=0.3$ and $P(B)=0.6$. Find

a) $P(A$ and not $B)$

b) P (neither A nor B).

PART – C

Answer **any six** questions : **(6 × 3 = 18)**

32) Show that the relation R in the set of real numbers R defined as $R=\{(a, b) : a \leq b\}$ is Reflexive and Transitive but not symmetric.

33) Write $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$, $|x|<a$ in the simplest form.

34) Express $A=\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

35) Differentiate $(\log x)^{\cos x}$, $x>0$ with respect to x.

36) Find $\frac{dy}{dx}$ if $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$.

37) Find two positive numbers x and y such that $x+y=60$ and xy^3 is maximum.

38) Find $\int x\tan^{-1}x\,dx$.

39) Find the equation of a curve passing through the point (0, 1) and whose differential equation is given by $\frac{dy}{dx}=y\tan x$ $\left(y\neq 0 \text{ and } 0\leq x<\frac{\pi}{2}\right)$.

40) If $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ and $\vec{a}$ is perpendicular to $(\vec{b}+\vec{c})$, $\vec{b}$ is perpendicular to $(\vec{c}+\vec{a})$ and $\vec{c}$ is perpendicular to $(\vec{a}+\vec{b})$ then find $|\vec{a}+\vec{b}+\vec{c}|$.

41) Find the area of a triangle having the points $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$ as its vertices.

42) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

PART – D

Answer **any four** questions : (4 × 5 = 20)

43) Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible. Find the inverse of f.

44) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

then compute $(A + B)$ and $(B - C)$. Also verify that $A + (B - C) = (A + B) - C$.

45) Solve the following system of Linear equations by matrix method :

$$x + y + z = 6$$
$$y + 3z = 11$$
$$x - 2y + z = 0.$$

46) If $y = Ae^{mx} + Be^{nx}$ then show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.

47) Find the integral of $\frac{1}{x^2 + a^2}$ with respect to x and hence find $\int \frac{dx}{x^2 + 2x + 2}$.

48) Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the method of integration.

49) Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$: $y=0$ when $x=1$.

50) Derive the equation of a line in space which passes through a given point and parallel to a given vector both in vector and Cartesian form.

PART – E

Answer the following questions :

51) a) Prove that $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x \cos x}dx$. (6)

OR

b) Solve the following Linear Programming Problem graphically :

Maximise $Z=4x+y$ .... (1)

Subject to the constraints

$x+y \leq 50$ .... (2)

$3x+y \leq 90$ .... (3)

$x \geq 0,\ y \geq 0$ .... (4)

52) a) Show that the matrix $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation $A^2-5A+7I=O$, where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation find A^{-1}. (4)

OR

b) Find the value of K so that the function f defined as

$$f(x)=\begin{cases} Kx+1, & \text{if} \quad x \leq \pi \\ \cos x, & \text{if} \quad x > \pi \end{cases}$$

is continuous at $x=\pi$.